## Genome Assembly

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## Sanger Sequencing

DNA Polymerase reads the templite strand and synthesizes a now second strand to match:


IF 5\% of the $T$ nucleotides are actually dideoxy $T$, then each strand will terminate when it gets a ddT on is growingend:
S. - TACGCG6TARCG6TATGTTCGRCCGITTAGCTACCGAT*

5* - TACGCGGTARCGGTATGTTCGACCGTTTAGCT*
5* - TACGCGGTARCGGTATGTTCGRCCGTTT*
5* - thcgcgatahcgatatgitccaccgtt*
$5^{*}$ - TACGCG6TARCGGTATGTTCGRCCGT*
5' - TACGCG6TRACGGTATGTT*
5* - TACGCGGTARCGGTATGT*
$5^{\circ}$ - thcgcggtancegtate
$5^{*}$ - TACGCGGTARCGGT*
$5^{*}$ - TRCGCG6T*

## Sanger Sequencing



Next, run a gel of the terminated fragments, differently coloured depending on the final nucleotide.

Read the sequence from the gel from the bottom to the top.

## Shotgun Sequencing

 -Break DNA into bacterial artificial chromosomes (BACs). -Map the BACs to the genome and obtain a tiling path. -Apply the shotgun method to each BAC.

## Shotgun Sequencing

## Whole Genome Shotgun Sequencing Method



GCATTPCGAGTTACCTGGACAACCAGTGGTACTGAGGACGCAAGAGGCTTGATTGGCCAATAATAGTATAT
Generate Finished Sequence

## Sequencing

- Since genomes cannot currently be sequenced in one run, they must be sequenced as fragments and reassembled.
- Now that sequencing is becoming fast and accurate, how do we assemble the reads as quickly?
- Algorithms needed to do this.


## Mapping-Based Assembly

- One of the prereadings gave an overview of mapping ("comparative") genome assembly.
- We will focus on de novo assembly in this lecture.


## Algorithms

- Shotgun sequencing assembly problem
- Find the shortest common superstring of a set of sequences.
- Given $n$ strings $\left\{s_{1}, s_{2}, \ldots, s_{n}\right\}$ find the shortest string $T$ such that every $\mathrm{s}_{\mathrm{i}}$ is a substring of T .
- This is NP-hard, problem becomes unsolvable when implemented with many reads.
- Approximation algorithm for this: the greedy algorithm.
- Perform pairwise comparisons on the reads.
- Pick the highest scoring pair and merge.
- Repeat until no more merges can be done


## Algorithms

- Shotgun sequencing assembly problem
- Greedy algorithms were the first successful assembly algorithm implemented.
- Because of the greedy algorithm's limitations, two other algorithms were derived.



## Algorithms

- Overlap-layout-consensus
- Algorithm based on graph theory
- A graph is constructed:
- nodes are reads, edges represent overlapping reads
- A contig (contiguous sequence) is a simple path (node visited at most once) in the graph



## Algorithms

- Overlap-layout-consensus
- An assembler builds the graph
- Output is a set of nonintersecting simple paths, each path being a contig.
- Ideally looking for a traversal of the graph (visits all nodes exactly once), a Hamiltonian path (also NP-hard).
- We'll come back to a simplification of this.


## Repeats

- There are often repeats in the sequence. Assembly algorithms should detect these during assembly, not after, to reduce incorrect reconstruction.
- avoid "over-collapsing" repeats
- Repeats can be detected statistically or using algorithms.


## Repeats

- Statistical detection
- Under the assumption that the genome is sampled equally at random, we see that if a certain sequence comes up more than others, it is likely part of repeat sequence.
- Not a very good method, samples are not uniformly distributed.
- Graph-based detection
- Finds repeats in complex parts of the graph constructed during the assembly process.
- Complex areas investigated and attempted to be resolved.


## Scaffolding

- Scaffolding groups contigs into subsets with known order and orientation.
- Nodes are contigs.
- Directed edge is between two nodes when paired-end tags bridge the gap between them.


Mate pairs orient contigs during assembly

Scaffold


## Scaffolding

- Three basic problems
- Find all connected components
- Find a consistent orientation for all nodes in the graph. Nodes have two types of edges
- Same orientation
- Different orientation
- Consistent orientation possible only if all undirected cycles have an even number of reversal edges.
- Optimization problem: find the smallest number of edges to be removed so that no cycle has an odd number of reversal edges
- Fit the edges on a line so the least number of constraints is invalidated. (NP-complete)
- Analogy
- Like taking a map, broken into pieces, and reassembling it using the cities spanning more than one piece to help determine boundaries.


## More Assembly Methods

- many graph-based methods exist:
- EULER assembler: previously discussed
- string graph: same graph, remove redundant edges, establish edge constraints (must balance flux at each node), find shortest walk... a very complicated algorithm, fails when a repeat is longer than a read
- all methods minimise size of genome, result in repeat "over-collapsing", assemblies can be improved


## Maximum Likelihood Assembly

- Medvedev \& Brudno changed the goal of assembly: don't want to minimize size of genome, but maximize its likelihood!
- Take advantage of the high coverage to estimate the copy number of each read
- Maximizing the likelihood can be considered a "mininum-cost bidirected flow" problem


## Bidirected Overlap Graph

- Edges in bidirected graphs have two orientations: one at each end
- Therefore, three types of edges:

- ***For a walk in a bidirected graph, for each vertex on that
 walk, the orientation of the edge entering the vertex must be opposite that of the edge
 leaving the vertex.


## Bidirected Overlap Graph

- Each vertex is a double-stranded read, edges represent read overlaps
- Three possible ways that two double-stranded reads can overlap (corresponds to the three types of edges)
- Given two reads, each read can be oriented to the left or to the right, giving three possible overlaps:



## Bidirected Overlap Graph

- A walk along this graph that visits every vertex at least once produces the original doublestranded genome (under the assumptions that the whole genome was covered by the reads, and that the reads are error-free)
- Overlap graph is constructed by placing an edge between two reads if they overlap by a minimum number of characters
- Then perform transitive edge reduction: remove overlaps covered by two shorter overlaps


## Chinese Postman Problem

- In a weighted graph, a "tour" is a walk that traverses every edge at least once. A "circuit" is a cyclical tour.
- Chinese postman problem is to find a minimum weight circuit.
- Eulerian circuit is a circuit traversing each edge exactly once (not always possible).


## Minimum-Cost Biflow Problem

- Set upper (u) and lower (I) flow bounds on each edge, cost (c) for each edge
- Flow function $f$ must obey the constraint for each edge e $l(e) \leq f(e) \leq u(e)$
- For each vertex, the incoming flow is balanced with the outgoing flow
- Objective: Find the flow that minimizes

$$
\sum c_{e} f(e)
$$

## Adjusted Minimum-Cost Biflow Problem

- Upper and lower flow bounds on vertices as well
- Accomplished by splitting every vertex $v$ into two: $v^{+}$ and $r$
- $r$ serves as the "incoming" vertex, and inherits v's incoming edges
- $v^{+}$serves as the "outgoing" vertex, and inherits v's outgoing edges
- Finally add one edge between $r$ and $v^{+}$and assign it the upper and lower flow bounds for $v$

becomes


## Build Graph Given Spectrum

- Nodes are all ( $k-l$ )-molecule sequences present in $k$-molecule-spectrum (two per $k$-molecule).
- Directed edges connect 5'-heavy (k-l)mer to 3'-heavy one in positive strands, opposite in negative strands.
- Edges unweighted (all have weight of I).


## Demonstration

- How to build graph given $k$-moleculespectrum.
\{ATT/AAT, TGC/GCA, GCC/GGC, CCA/TGG, CAA/TTG, AAC/GTT\}



## Demonstration

- Convert to bidirected de Bruijn graph:



## Demonstration

- Now find circular walk in this graph (includes all edges)
- While walking, append each node minus the overlap to some string.



## Conversion to Algorithm

- This procedure is trivial to do manually for small graphs, but imagine a graph of tens of thousands of 50-mers, for example. An efficient algorithm is needed.
- We already have algorithms for shortest path, but it needs to be modified for this special case.


## Adjustments

- Supersource and supersink added to convert flow problem into circulation problem
- Each vertex has a lower bound of I, since each read must appear in the finished genome at least once
- Edge bounds are set to 0 (lower bound) and infinity (upper bound)
- Put prohibitively large cost on the edge leading from the supersource and the edge leading to the supersink to ensure that the assembly uses the smallest number of contigs possible
- Flow through each vertex represents number of times it appears in the assembled genome


## Methods: Maximizing the Global ReadCount Likelihood

- Start with the probability of a $k$-mer $i$ being sampled a certain number of times from a genome $G$
- Let $N(G)$ be the length of the genome assembly of $G$, and let $g_{i}$ be the frequency of $i$ in $G$
- Under the assumption of uniform sampling, the probability of sampling $i$ is $g_{i} / N(G)$
- Let $X_{i}$ be the random variable that represents the number of trials whose outcome is $i$
- Each random variable for every possible $k$-mer has a binomial distribution. Their joint distribution is the following multinomial distribution:

$$
P\left[X_{1}=x_{1}, X_{2}=x_{2}, \ldots, X_{4^{k}}=x_{4^{k}}\right]=\frac{n!}{\prod x_{i}} \prod_{i}\left(\frac{g_{i}}{N(G)}\right)^{x_{i}}
$$

## Putting It Together

- Build the bidirected overlap graph.
- Perform transitive edge reduction (remove redundant overlaps).
- Add supersource and supersink and appropriate weightings to the graph.
- Solve biflow problem (using likelihoods).
- very confusing portion of algorithm
- Solution gives collection of walks, representing contigs.
- Assemble contigs using paired-end read data.


## Likelihood Algorithm

- From this, derive the global read-count likelihood, the likelihood of $k$-mer distributions $\left(g_{i}\right)$ given the sampling outcomes $\left(x_{i}\right)$ :

$$
L\left[g_{1}, \ldots, g_{4^{k}} \mid x_{1}, \ldots, x_{4^{k}}\right]=\frac{n!}{\prod x_{i}} \prod\left(\frac{g_{i}}{N(G)}\right)^{x_{i}}
$$

- Goal is to maximize $L$, or, equivalently, minimize the negative log of $L$
- To translate this problem into a convex mincost biflow problem, we need convex functions for each $k$-mer $c_{i}$ s.t. $-\log L=\sum c_{i}\left(g_{i}\right)$
- Problem: the $X_{i}$ random variables are not independent...


## Likelihood Algorithm

- ... unless the number of trials approaches infinity
- The number of trials is usually large enough to warrant the approximation of the multinomial distribution as the product of the binomial distributions for each $X_{i}$
- In this binomial approximation, genome length $N(G)$ is constant, and independent of the sampling frequencies
- Therefore, use $N$ instead, which is the actual length of the genome $G$


## Likelihood Algorithm

- New approximation of $L$ :

- Now $-\log L=K \sum c_{i}\left(g_{i}\right)$
- And $c_{i}\left(g_{i}\right)=-\left(x_{i} \log g_{i}\right)-\left(n-x_{i}\right) \log \left(N-g_{i}\right)$
- $c_{i}$ is used as the convex functions for the vertices of the min-cost biflow graph described earlier


## Convert Flow to Contigs

- Decompose flow into a collection of walks, which translates into assembled contigs
- Graph is first simplified by removing all edges with a flow of zero
- Additional simplifications possible by removing vertices $v$ where:
- There is exactly one edge going into $v$ and one edge leading out of $v$, and the flow on both edges is the same

- Vertices where there is also a loop with the same flow as the other two edges, and

- Split and join vertices, where the flow on the in edges is the same as those of the out-edges



## (Potentially Problematic) Assumptions

- First major assumption: Reads are errorfree
- can be overcome with higher coverage
- Second major assumption: Uniform sampling of all genomic regions
- certain portions of the genome are actually easier to sample than others


## Assembly Quality

- Assessing Assembly Quality
- misassembly correction is expensive
- some assemblers have a simple quality-control method that does not capture larger errors
- test assembly software if we know a complete sequence (artificial or real)
- measures of quality: number and sizes of contigs
- under assumption that having few large contigs is better than many small ones.
- only partially true, because there are less gaps in the former, but, does not account for the possibility of misassemblies


## Questions?

