# Dynamic Programming and 

## Pairwise Sequence Alignment

Zahra Ebrahim zadeh
z.ebrahimzadeh@utoronto.ca


## General Outline

-Importance of Sequence Alignment
-Pairwise Sequence Alignment
-Dynamic Programming in Pairwise Sequence Alignment
-Types of Pairwise Sequence Alignment

## Importance of Sequence Alignment

- To identify regions of similarity : indicating functional and structural relationship
- To determine homology


# What is pairwise sequence alignment? 

```
IFCZ: S PQ LEELITKVSKAHQ ETFP - . . . - S LCQLGK - -
3U9Q: SADLRALAKHLYDSYIKSFPLTKAKARAILTGKTT
```

The process of lining up two nucleotide or amino acid sequences to obtain highest score of similarity for the purpose of assessing the degree of similarity and possibility of homology

## Alignment Operation

Transforming one string into the other by a series of edit operations on individual characters

Edit operations was first introduced in the edit distance concept by Levenshtein 1966.

- Insertion (I) of a character into the first string
- Deletion (D) of a character from the first string
- Substitution $(\mathrm{S})$ of a character in the first string that mismatches the aligned character in the second string
- Match (M) of a character in the first strings with a character in the second string


## Alignment Operation

Transforming one string into the other by a series of edit operations on individual characters

Edit operations was first introduced in the edit distance concept by Levenshtein 1966.

- Insertion (I) of a character into the first string
- Deletion (D) of a character from the first string
- Substitution $(\mathrm{S})$ of a character in the first string that mismatches the aligned character in the second string
- Match (M) of a character in the first strings with a character in the second string

$$
\text { Example: V = THISLINE and } W=\text { ISALIGNED }
$$



## Difficulties in measuring sequence similarities

- Sequences usually differ in length
- Sequences may only have very small region of similarity
- Some substitution are more likely than others


## Efficient way to find a best alignment

Consider aligning two sequences $V=\left(v_{1} v_{2} \ldots v_{n}\right)$ and $W=\left(w_{1} w_{2} \ldots w_{m}\right)$.
Can we use Brute-Force method to create all the possible alignment, and then find the alignment with highest similarity score?

## Efficient way to find a best alignment

Consider aligning two sequences $V=\left(v_{1} v_{2} \ldots v_{n}\right)$ and $W=\left(w_{1} w_{2} \ldots w_{m}\right)$.
Can we use Brute-Force method to create all the possible alignment, and then find the alignment with highest similarity score?

This takes exponential time!

## Efficient way to find a best alignment

Consider aligning two sequences $V=\left(v_{1} v_{2} \ldots v_{n}\right)$ and $W=\left(w_{1} w_{2} \ldots w_{m}\right)$.
Can we use Brute-Force method to create all the possible alignment, and then find the alignment with highest similarity score?

This takes exponential time!

Dynamic Programming finds the optimal (best) alignment efficiently.

## Problems Solvable by Dynamic Programming



Gene Recognition
From: pcp.oxfordjournals.org

## Dynamic Programming (DP)

A method for efficiently solving optimization problems which have overlapping subproblems

## Property of DP problems

- Have overlapping subproblems
- Have optimal solutions to subproblems
- Can be represented in recurrence relation
- Are context-independent
e.g. In sequence alignment, quantifying similarity is only based on pairs of residues.

Similarity is independent of environment of residues we align.

## Classes of Pairwise Alignment:

## Classes of Pairwise Alignment:

## I. Global Alignment

## Classes of Pairwise Alignment:

I. Global Alignment

Best match in the entire sequences

$$
\begin{array}{llllllllllll}
A & T & A & C & A & G & C & G & G & T & C & T \\
A & - & - & C & A & G & - & G & G & T & - & T
\end{array}
$$

## Classes of Pairwise Alignment:

I. Global Alignment
II. Local Alignment

Best match in the entire sequences

$$
\begin{array}{cccccccccccc}
A & T & A & C & A & G & C & G & G & T & C & T \\
A & - & - & C & A & G & - & G & G & T & - & T
\end{array}
$$

## Classes of Pairwise Alignment:

I. Global Alignment
II. Local Alignment

Best match in the entire sequences

$$
\begin{array}{cccccccccccc}
A & T & A & C & A & G & C & G & G & T & C & T \\
A & - & - & C & A & G & - & G & G & T & - & T
\end{array}
$$

Best subsequence match

$$
\begin{array}{lllllllllllll}
A & T & A & C & A & G & C & G & G & T & - & C & T \\
- & - & A & C & A & G & - & G & G & T & T & - & -
\end{array}
$$

## Classes of Pairwise Alignment:

## I. Global Alignment

II. Local Alignment

Best subsequence match

$$
\begin{array}{lllllllllllll}
A & T & A & C & A & G & C & G & G & T & - & C & T \\
- & - & A & C & A & G & - & G & G & T & T & - & -
\end{array}
$$

III. Semi-Global Alignment
"Glocal" Alignment

Best match in the entire sequences

| $A$ | $T$ | $A$ | $C$ | $A$ | $G$ | $C$ | $G$ | $G$ | $T$ | $C$ | $T$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | - | - | $C$ | $A$ | $G$ | - | $G$ | $G$ | $T$ | - | $T$ |

## Classes of Pairwise Alignment:

## I. Global Alignment

II. Local Alignment

Best match in the entire sequences

$$
\begin{array}{cccccccccccc}
A & T & A & C & A & G & C & G & G & T & C & T \\
A & - & - & C & A & G & - & G & G & T & - & T
\end{array}
$$

Best subsequence match

$$
\begin{array}{lllllllllllll}
A & T & A & C & A & G & C & G & G & T & - & C & T \\
- & - & A & C & A & G & - & G & G & T & T & - & -
\end{array}
$$

III. Semi-Global Alignment
"Glocal" Alignment

Best match without penalizing gaps on the ends of the alignment
$\begin{array}{llllllllllllll}\mathrm{T} & \mathrm{C} & \mathrm{T} & \mathrm{G} & \mathrm{T} & - & A & C & C & G & T & G & - & - \\ - & - & - & G & T & T & A & C & C & A & T & G & C & C\end{array}$

## Global Alignment

- Assumes the sequences are similar over the length of one another
- The alignment attempts to match them to each other from end to end

```
IFCZ: S PQLEELITKVSKAHQETFP-....-SLCQLGK - -
3U9Q: SADLRALAKHLYDSYIKSFPLTKAKARAILTGKTT
```



Optimal global alignments are produced using Needleman-Wunsch Algorithm

## Needleman-Wunsch Algorithm

A dynamic programming algorithm for optimal global alignment

Given:

$$
\begin{aligned}
& \text { Two sequences } V=\left(v_{1} v_{2} \ldots v_{n}\right) \text { and } W=\left(w_{1} w_{2} \ldots w_{m}\right) . \\
& (|V|=n \text { and }|W|=m)
\end{aligned}
$$

Goal:
Find the best scoring alignment in which all residues of both sequences are included. The score is usually a measure of similarity.

Requirement:

- A matrix NW of optimal scores of subsequence alignments. NW has size $(\mathrm{n}+1) \times(\mathrm{m}+1)$.
- Scoring matrix
- Defined gap penalty


## Scoring matrix

represents a specific model of similarity to be applied in aligning two residues

- Matrix of numbers that quantify the similarity between residues
- To produce good alignment, the choice of a right scoring matrix is important
- Common scoring matrices:
- Identity Matrix
- Genetic Code Matrix
- PAM Matrices
- BLOSUM Matrices
- Protein sequences are frequently aligned using PAM or BLOSUM matrices that reflect the frequency with

BLOSUM62
 which a amino acid replaces another amino acid in evolutionarily related sequences.

- Some amino acid substitutions are commonly found throughout the process of molecular evolution while others are rare.
e.g. the probability that Ser mutates into Phe is $\sim$ three times greater than the probability that Trp mutates into Phe


## Gap Penalty

a score for gap between the residues of sequences in sequence alignment

Gaps inserted in a sequence to maximize similarity with another, require a scoring penalty.

Gap opening penalty: penalty for starting a new gap in a sequence.

Gap extension penalty: penalty for adding gaps to an existing gap.

## Common Gap Models:

- Constant gap: g = - (gap opening penalty)
- Linear gap: $\mathrm{g}\left(\mathrm{n}_{\mathrm{gap}}\right)=-\mathrm{n}_{\mathrm{gap}}$. (gap extension penalty $)$
- Affine gap: $\mathrm{w}\left(\mathrm{n}_{\text {gap }}\right)=-($ gap opening penalty $)-\left[\mathrm{n}_{\text {gap }} .(\right.$ gap extension penalty $\left.)\right]=\mathrm{g}+\mathrm{g}\left(\mathrm{n}_{\text {gap }}\right)$

Affine gap model is used extensively in biology domain.

## Needleman-Wunsch Algorithm (Cont.)

## A dynamic programming algorithm for optimal global alignment

Given:

$$
\begin{aligned}
& \text { Two sequences } V=\left(v_{1} v_{2} \ldots v_{n}\right) \text { and } W=\left(w_{1} w_{2} \ldots w_{m}\right) . \\
& (|V|=n \text { and }|W|=m)
\end{aligned}
$$

Goal:
Find the best scoring alignment in which all residues of both sequences are included. The score is usually a measure of similarity.

Requirement:

- A matrix NW of optimal scores of subsequence alignments.

NW has size $(\mathrm{n}+1) \times(\mathrm{m}+1)$.

- Score matrix
- Defined gap penalty


## Needleman-Wunsch Algorithm (Cont.)

## Calculation

Let $\mathrm{NW}(\mathrm{i}, \mathrm{j})$ be the optimal alignment score of aligning $\mathrm{V}[1 . . . \mathrm{i}]$ and $\mathrm{W}[1 \ldots \mathrm{j}]$

|  |  | W/ | ... | w ${ }_{\text {j }}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  |  |  |  |  |
| VI |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| $\mathrm{V}_{\mathrm{i}}$ |  |  |  |  |  |  |
| : |  |  |  |  |  |  |
| $V_{n}$ |  |  |  |  |  | $\begin{gathered} \text { Optimal } \\ \text { alignment } \\ \text { score } \end{gathered}$ |

## Needleman-Wunsch Algorithm (Cont.)

## Calculation

Let $\mathrm{NW}(\mathrm{i}, \mathrm{j})$ be the optimal alignment score of aligning $\mathrm{V}[1 . . . \mathrm{i}]$ and $\mathrm{W}[1 \ldots \mathrm{j}]$

|  |  | WI | ... | W | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  |  |  |  |  |
| VI |  |  |  |  |  |  |
| : |  |  |  |  |  |  |
| V |  |  |  |  |  |  |
| : |  |  |  |  |  |  |
| $V_{n}$ |  |  |  |  |  | Optimal alignment |

Base case: $\left\{\begin{array}{l}N W(0,0)=0 \\ N W(0, j)=N W(0, j-1)+g \\ N W(i, 0)=N W(i-1,0)+g\end{array}\right.$
For linear gap penalty model

Recurrence: $\quad N W(i, j)=\max \begin{cases}N W(i-1, j-1)+s\left(v_{i}, w_{j}\right) & \text { match/mismatch } \\ N W(i-1, j)+g & \text { delete } \\ N W(i, j-1)+g & \text { insert }\end{cases}$

## Dynamic Programming Approach

Construct an optimal alignment between two subsequences ( $\mathrm{v}_{1} \mathrm{v}_{2} \ldots \mathrm{v}_{\mathrm{i}}$ ) and ( $\mathrm{W}_{1} \mathrm{~W}_{2} \ldots \mathrm{~W}_{\mathrm{i}}$ ), (Where $0 \leq i \leq n$ and $0 \leq j \leq m$ ), by considering the three cases:
(I) The optimal alignment of $\mathrm{v}_{\mathrm{I}}, \ldots, \mathrm{v}_{\mathrm{i}-\mathrm{l}}$ with $\mathrm{w}_{\mathrm{l}}, \ldots \mathrm{w}_{\mathrm{j}-1}$, extended by the match between $v_{i}$ and $w_{j}$.
(II)The optimal alignment of $\mathrm{v}_{\mathrm{l}}, \ldots, \mathrm{v}_{\mathrm{i}-\mathrm{l}}$ with $\mathrm{w}_{\mathrm{l}}, \ldots \mathrm{w}_{\mathrm{j}}$, extended by matching a gap character with $\mathrm{v}_{\mathrm{i}}$.
(III)The optimal alignment of $\mathrm{v}_{1}, \ldots, \mathrm{v}_{\mathrm{i}}$ with $\mathrm{w}_{\mathrm{l}}, \ldots \mathrm{w}_{\mathrm{j}-\mathrm{l}}$, extended by matching $\mathrm{w}_{\mathrm{j}}$ with a gap character "-".

Store these optimal scores of subsequence alignments in a matrix of size $(n+1) \times(m+1)$.

## Needleman-Wunsch Algorithm (Cont.)

Traceback

To recover the optimal alignment, arrows indicating forward calculation paths, are placed in each entry.

|  |  | WI | $\bullet \bullet \bullet$ | Wj | $\bullet \bullet$ | $W_{\text {m }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  |  |  |  |  |
| VI |  |  |  |  |  |  |
| : |  |  |  |  |  |  |
| Vi |  |  |  |  |  |  |
| : |  |  |  |  |  |  |
| V |  |  |  |  |  | Optimal ignment score |

$$
\begin{aligned}
& \searrow: N W(i, j)=N W(i-1, j-1)+s\left(v_{i}, w_{j}\right) \\
& \downarrow: N W(i, j)=N W(i-1, j)+\text { gap } \\
& \longrightarrow: N W(i, j)=N W(i, j-1)+\text { gap }
\end{aligned}
$$

Determine alignment from the end of the sequences


## Needleman-Wunsch Algorithm (Cont.)

Example

Optimal global alignment of $V=$ THISLINE and $W=$ ISALIGNED with gap $=-4 n_{g a p}$, score matrix $=$ BLOSUM62

|  |  | I | S | A | L | I | G | N | E | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 04 | 4 | -8 | -12 | $\rightarrow-16$ | $\rightarrow 20$ | $\rightarrow 24$ | $\rightarrow-28$ | $\rightarrow 32$ | -36 |
| T | k | -1 | -3 | -7 | -1 | -1 | $\rightarrow-19$ | $\rightarrow-23$ | $\rightarrow$-27 | $\rightarrow$-31 |
| H | 8 | -5 | -2 | ${ }^{3}-5$ | -9 | -13 | -17 | $\rightarrow-18$ | -22 | -26 |
| I | 12 |  | -6 | -3 | -3 | -5 | -9 | -13 | -17 | -2 |
| S | $-16$ | -8 |  |  | -5 | -5 | -5 | -8 | -12 | -1 |
| L | -20 | -12 | -4 | -1 |  | -3 | -7 | -8 | -11 | -1 |
| I | $-24$ | -16 | -8 | -5 | 1 |  | 0 | -4 | -8 | -12 |
| N | $-28$ | -20 | -12 | -9 | -3 | 0 | 4 |  | 2 | -2 |
| E | $\checkmark 32$ | -24 | -16 | -13 | -7 | -4 | 0 | 4 | 1 | 7 |

From: Understanding Bioinformatics by Zvelebil, Baum

| V: | T | H | I | S | - | L | I | - | N | E | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| W: | - | - | I | S | A | L | I | G | N | E | D |

## Needleman-Wunsch Algorithm Efficiency

Runtime: $\mathrm{O}(\mathrm{nm})$

## Why?

Space: $\mathrm{O}(\mathrm{nm})$

# Needleman-Wunsch Algorithm 

for any gap penalty models
Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

## Needleman-Wunsch Algorithm

for any gap penalty models
Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

|  |  | WI | ... | $\mathrm{w}_{\mathrm{j}}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  |  |  | (i) |  |
| vi |  |  |  |  | gaps |  |
| : |  |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{i}}$ | + (i) gap |  | P |  |  |  |
| : |  |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{n}}$ |  |  |  |  |  | $\begin{aligned} & \text { Optimal } \\ & \text { alignment } \\ & \text { score } \\ & \hline \end{aligned}$ |

## Needleman-Wunsch Algorithm

## for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?


$$
N W(i, j)=\max \begin{cases}N W(i-1, j-1)+s\left(v_{i}, w_{j}\right) & \text { match/mismatch } \\ {\left[N W\left(i-n_{g a p}, j\right)+w\left(n_{g a p 1}\right)\right]_{1 \leq n_{\text {gap }} \leq i} \leq} & \text { delete } \\ {\left[N W\left(i, j-n_{g a p 2}\right)+w\left(n_{g a p 2}\right)\right]_{1 \leq n_{g a p} \leq j} \leq} & \text { insert }\end{cases}
$$

## Needleman-Wunsch Algorithm

## for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

|  |  | w 1 | ... | $\mathrm{w}_{\mathrm{i}}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  |  |  | +(i) |  |
| v |  |  |  |  | gaps |  |
| : |  |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{i}}$ | + (i) gap | +( | $+$ |  |  |  |
| : |  |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{n}}$ |  |  |  |  |  |  |

$$
N W(i, j)=\max \begin{cases}N W(i-1, j-1)+s\left(v_{i}, w_{j}\right) & \text { match/mismatch } \\ {\left[N W\left(i-n_{g a p}, j\right)+w\left(n_{\text {gap } 1}\right)\right]_{1 \leq n_{\text {gap }} \leq i}} & \text { delete } \\ {\left[N W\left(i, j-n_{\text {gap } 2}\right)+w\left(n_{\text {gap } 2}\right)\right]_{1 \leq n_{g a p 2} \leq j}} & \text { insert }\end{cases}
$$

What is the runtime? or space?

## Needleman-Wunsch Algorithm

## for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

|  |  | w 1 | ... | $\mathrm{w}_{\mathrm{i}}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  |  |  | +(i) |  |
| v |  |  |  |  | gaps |  |
| : |  |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{i}}$ | + (i) gap | +( | $+$ |  |  |  |
| : |  |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{n}}$ |  |  |  |  |  |  |

$$
N W(i, j)=\max \begin{cases}N W(i-1, j-1)+s\left(v_{i}, w_{j}\right) & \text { match/mismatch } \\ {\left[N W\left(i-n_{g a p}, j\right)+w\left(n_{\text {gap } 1}\right)\right]_{1 \leq n_{\text {gap }} \leq i}} & \text { delete } \\ {\left[N W\left(i, j-n_{\text {gap } 2}\right)+w\left(n_{\text {gap } 2}\right)\right]_{1 \leq n_{g a p 2} \leq j}} & \text { insert }\end{cases}
$$

What is the runtime? or space? $\mathrm{O}\left(\mathrm{mn}^{2}\right)$ where $\mathrm{n}>\mathrm{m}$

## Local Alignment

finds the most similar regions of a nucleotide or amino acid sequence ignoring other segments of the sequences

Local alignment programs are useful for detecting shared domains in multi-domain proteins.

```
IFCZ: ATKC I IK I V E FAKR L PG FTGLS IAACLD I LMLRIC
3U9Q:S V EAVQ E I T EYAKS I PG FVNLDLNDQVTLLLKYGVH
```



IFCZ and 3 U 9 Q superimposed

## Optimal local alignments are produced using Smith-Waterman Algorithm

## Smith-Waterman Algorithm

A dynamic programming algorithm for optimal local alignment

## Given:

Two sequences $V=\left(v_{1} v_{2} \ldots v_{n}\right)$ and $W=\left(w_{1} w_{2} \ldots w_{m}\right)$.
$(|\mathrm{V}|=\mathrm{n}$ and $|\mathrm{W}|=\mathrm{m})$

Goal:
Find the highest scoring alignment for best subsequence match. The score is usually a measure of similarity.

Requirement:

- A matrix SW of optimal scores of subsequence alignments. SW has size $(\mathrm{n}+1) \times(\mathrm{m}+1)$.
- Score matrix
- Defined gap penalty


## Smith-Waterman Algorithm (Cont.)

## Smith-Waterman Algorithm (Cont.)

|  |  | W/ | ... | $\mathrm{w}_{\mathrm{i}}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 |
| VI | 0 |  |  |  |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{i}}$ | 0 |  |  |  |  |  |
| : | 0 |  |  |  |  |  |
| V | 0 |  |  |  |  |  |

## Smith-Waterman Algorithm (Cont.)

|  |  | WI | ... | w ${ }_{\text {j }}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 |
| v ı | 0 |  |  |  |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{V}_{\mathrm{i}}$ | 0 |  |  |  |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{V}_{\mathrm{n}}$ | 0 |  |  |  |  |  |

Optimal alignment score $=\max _{0 \leq i \leq n, 0 \leq j \leq m}\{S W(i, j)\}$

## Smith-Waterman Algorithm (Cont.)

|  |  | W/ | ... | $w_{i}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 |
| VI | 0 |  |  |  |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{v}^{\text {i }}$ | 0 |  | (II) | $\xrightarrow{ }$ |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{n}}$ | 0 |  |  |  |  |  |

Optimal alignment score $=\max _{0 \leq i \leq n, 0 \leq j \leq m}\{S W(i, j)\}$

For linear gap penalty model
Base case: $\operatorname{SW}(\mathrm{i}, \mathrm{j})=0$ where $\mathrm{i}=0$ or $\mathrm{j}=0$
Recurrence: $\quad S W(i, j)=\max \begin{cases}0 & \begin{array}{ll}\text { align empty strings } \\ S W(i-1, j-1)+s\left(v_{i}, w_{j}\right) & \text { match/mismatch }\end{array} \\ S W(i-1, j)+g & \text { delete } \\ S W(i, j-1)+g & \text { insert }\end{cases}$

## Smith-Waterman Algorithm (Cont.)

Example

Local alignment of $\mathrm{V}=$ THISLINE, $\mathrm{W}=$ ISALIGNED with gap $=-4 \mathrm{n}_{\text {gap }}$, score matrix $=$ BLOSUM62

|  |  | $\mathbf{I}$ | $\mathbf{S}$ | $\mathbf{A}$ | $\mathbf{L}$ | $\mathbf{I}$ | $\mathbf{G}$ | $\mathbf{N}$ | $\mathbf{E}$ | $\mathbf{D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{T}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{H}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{I}$ | $\mathbf{0}$ | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{2}$ | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{S}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{8}$ | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{4}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{L}$ | $\mathbf{0}$ | $\mathbf{2}$ | $\mathbf{4}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{I}$ | $\mathbf{0}$ | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{3}$ | $\mathbf{9}$ | $\mathbf{1 2}$ | $\mathbf{0}$ | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{N}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{5}$ | $\mathbf{1}$ | $\mathbf{5}$ | $\mathbf{8}$ | $\mathbf{1 2}$ | $\mathbf{1 4}$ | $\mathbf{1 0}$ | $\mathbf{6}$ |
| $\mathbf{E}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{4}$ | $\mathbf{1}$ | $\mathbf{4}$ | $\mathbf{8}$ | $\mathbf{1 2}$ | $\mathbf{1 9}$ | $\mathbf{1 5}$ |

From: Understanding Bioinformatics by Zvelebil, Baum
Traceback starts at entry containing the optimal alignment score.

| $\mathrm{V}:$ | $\mathbf{I}$ | $\mathbf{S}$ | - | $\mathbf{L}$ | $\mathbf{I}$ | - | $\mathbf{N}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{W}:$ | $\mathbf{I}$ | $\mathbf{S}$ | $\mathbf{A}$ | $\mathbf{L}$ | $\mathbf{I}$ | $\mathbf{G}$ | $\mathbf{N}$ | $\mathbf{E}$ |

## Smith-Waterman Algorithm Efficiency

Runtime: $\mathrm{O}(\mathrm{nm})$

## Why?

Space: $\mathrm{O}(\mathrm{nm})$

## Which alignment to use?

Example I. Overlap detection:Aligning exon to a gene sequence
$\mathrm{v}=\mathrm{ATCCGAACATCCAATCGAAGC} \quad \mathrm{W}=A G C A T G C A A T$
Aligning scores: match $=2$, gap $=-1$ mismatch $=-2$

## Which alignment to use?

Example I. Overlap detection:Aligning exon to a gene sequence
$\mathrm{v}=$ ATCCGAACATCCAATCGAAGC $\mathrm{W}=$ AGCATGCAAT
Aligning scores: match $=2$, gap $=-I$ mismatch $=-2$

Global alignment?

## Which alignment to use?

Example I. Overlap detection:Aligning exon to a gene sequence
$\mathrm{v}=\mathrm{ATCCGAACATCCAATCGAAGC} \quad \mathrm{W}=A G C A T G C A A T$
Aligning scores: match $=2$, gap $=-I$ mismatch $=-2$

Global alignment?

$$
\begin{aligned}
& \text { A T C C G A A C A T C C A A T C G A A G C } \\
& \text { A- - G--C A T G C A A T - - - - } \\
& \text { Score }=2(9)-I(11)-2(1)=5
\end{aligned}
$$

## Which alignment to use?

Example I. Overlap detection:Aligning exon to a gene sequence
$\mathrm{v}=\mathrm{ATCCGAACATCCAATCGAAGC} \quad \mathrm{W}=A G C A T G C A A T$
Aligning scores: match $=2$, gap $=-I$ mismatch $=-2$

Global alignment?

$$
\begin{aligned}
& \text { A T C C G A A C A T C C A A T C G A A G C } \\
& \text { A- - G--C A T G C A A T - - - } \\
& \text { Score }=2(9)-I(I I)-2(1)=5
\end{aligned}
$$

Local alignment?

## Which alignment to use?

Example I. Overlap detection:Aligning exon to a gene sequence
$\mathrm{v}=\mathrm{ATCCGAACATCCAATCGAAGC} \quad \mathrm{W}=A G C A T G C A A T$
Aligning scores: match $=2$, gap $=-I$ mismatch $=-2$

Global alignment?

> A T C C G A A C A T C C A A T C G A A G C A- - G--C A T G C A A T - - -
> Score $=2(9)-I(I I)-2(1)=5$

Local alignment?

$$
\begin{aligned}
& \text { C A T C C A A T } \\
& \text { C A T G C A A T } \\
& \text { Score }=2(7)-2(1)=12
\end{aligned}
$$

## Which alignment to use?

Example I. Overlap detection:Aligning exon to a gene sequence
$\mathrm{V}=$ ATCCGAACATCCAATCGAAGC $\quad \mathrm{W}=$ AGCATGCAAT
Aligning scores: match $=2$, gap $=-I$ mismatch $=-2$

Global alignment?

> A T C C G A A C A T C C A A T C G A A G C A - - G--C A T G C A A T - - -
> Score $=2(9)-1(11)-2(1)=5$

Local alignment?

$$
\begin{aligned}
& \text { C A T C C A A T } \\
& \text { C A T G C A A T } \\
& \text { Score }=2(7)-2(1)=12
\end{aligned}
$$

Where was the overlap exactly?


## Which alignment to use?

What if avoid penalizing the gaps at the beginning and /or the end of an alignment?

$$
\begin{aligned}
& \text { A T C C GA - CATCCAATCGAAGC } \\
& ----A G C A T G C A A T-\cdots \\
& \text { Score }=2(8)-I(I)-2(I)=13
\end{aligned}
$$

Spaces in front or end of the exon might be UTR, introns, or enhancer and etc.
Thus these gaps should not be penalized.

## Which alignment to use?

What if avoid penalizing the gaps at the beginning and /or the end of an alignment?

$$
\begin{aligned}
& \text { ATCCGA-CATCCAATCGAAGC } \\
& -\cdots-A G C A T G C A A T-\cdots-\cdots \\
& \text { Score }=2(8)-1(1)-2(I)=13
\end{aligned}
$$

Spaces in front or end of the exon might be UTR, introns, or enhancer and etc. Thus these gaps should not be penalized.

Semi-global alignment. Globally aligning the two sequence but ignoring penalizing gaps at both ends of a sequence.

## Which alignment to use? (Cont.)

Example 2. Overlap detection: Sequence assembly:
v = ACCTCACGATCCGA
W =TCAACGATCACCGCA
$\cdots-\cdots--\operatorname{ACCTCACGATCCGA}$
TCAACGATCACCGCA $-\cdots \cdots$

Semi-global alignment. Globally aligning the two sequence but ignoring penalizing the starting gaps of a sequence and the trailing gaps of the other sequence.

## Semi-Global Alignment

finds optimal alignment without penalizing gaps on the ends of the alignment
How to perform semi-global alignment?
Modify the basic Needleman-Wunsch algorithm:
Set the first row and first column of the DP matrix to 0 .

|  |  | W/ | ... | $\mathrm{w}_{\mathrm{i}}$ | ... | $\mathrm{W}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 |
| VI | 0 |  |  |  |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{i}}$ | 0 |  | (II) -+ | $\xrightarrow{\square}$ |  |  |
| : | 0 |  |  |  |  |  |
| $\mathrm{v}_{\mathrm{n}}$ | 0 |  |  |  |  |  |

Optimal alignment score $=\max \left(\right.$ row $_{\mathrm{n}}$, column $_{\mathrm{m}}$ )
Traceback starts at entry containing the optimal alignment score and ends at the first row or the first column.

## Versatility of DP Algorithm

- Memory usage can be optimized
- Runtime can be improved


## Versatility of DP Algorithm

- Memory usage can be optimized
- Runtime can be improved
- Heuristically can improve the runtime:
- FASTA
- BLAST


## References

- Gusfield D. Algorithms on strings, trees and sequences. The press syndicate of the University of Cambridge; 1997. p.215-244.
- Zvelebil M, Baum JO. Understanding Bioinformatics. New York: Garland Science; 2008. p. 126-137.
- Steipe B. Homology I: Principles. BCH441 Lecture Fall 2010.
- Jones NC, Pevzner AP. An Introduction to Bioinformatics Algorithms. London: The MIT press; 2004. p. 177-181
- Wing-kin S. Algorithms in Bioinformatics: A practical introduction. London: CRC Press; 2010. p. 32-42.
- Setubal J, Meidanis J. Introduction to Computational Molecular Biology. London: The MIT press. 2007. Chapter 3.

Any Question?

