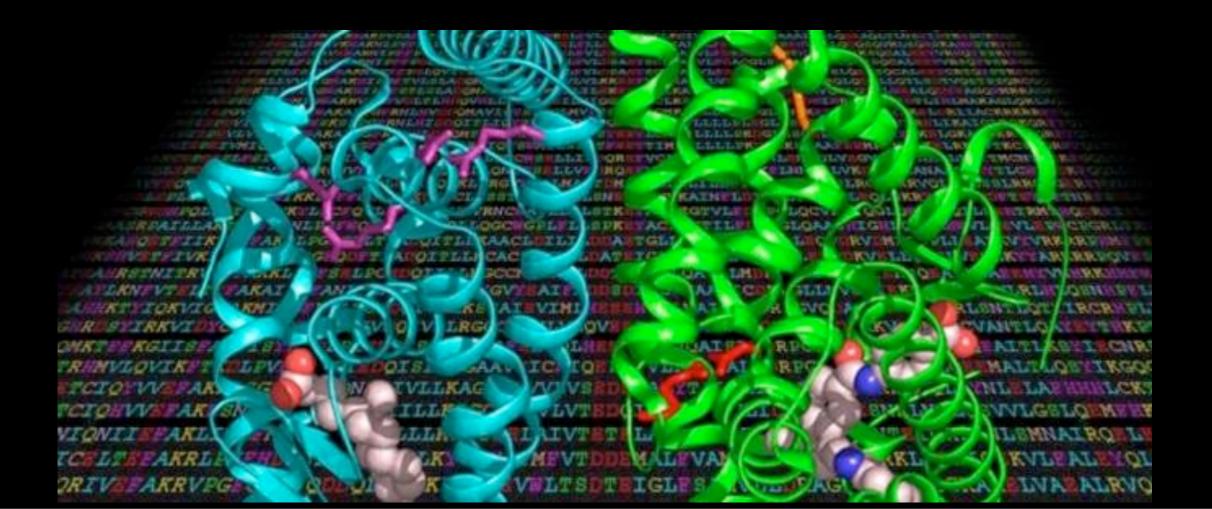
## Dynamic Programming and Pairwise Sequence Alignment

Zahra Ebrahim zadeh z.ebrahimzadeh@utoronto.ca



#### General Outline

- Importance of Sequence Alignment
- Pairwise Sequence Alignment
- Dynamic Programming in Pairwise Sequence Alignment
- Types of Pairwise Sequence Alignment

#### Importance of Sequence Alignment

#### • To identify **regions of similarity** :

indicating functional and structural relationship

#### • To determine **homology**

#### What is pairwise sequence alignment?

#### IFCZ: S PQ L E E L I T K V S K A H Q E T F P - - - - S L C Q L G K - -3U9Q: S A D L R A L A K H L Y D S Y I K S F P L T K A K A R A I L T G K T T

The process of lining up two nucleotide or amino acid sequences to obtain highest score of similarity for the purpose of assessing the degree of similarity and possibility of homology

## **Alignment Operation**

Transforming one string into the other by a series of edit operations on individual characters

Edit operations was first introduced in the edit distance concept by Levenshtein 1966.

- Insertion (I) of a character into the first string
- Deletion (D) of a character from the first string
- Substitution (S) of a character in the first string that mismatches the aligned character in the second string
- Match (M) of a character in the first strings with a character in the second string

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- Substitution (S) of a character in the first string that mismatches the aligned character in the second string
- Match (M) of a character in the first strings with a character in the second string

Example: V = THISLINE and W = ISALIGNED

Difficulties in measuring sequence similarities

• Sequences usually differ in length

• Sequences may only have very small region of similarity

• Some substitution are more likely than others

#### Efficient way to find a best alignment

Consider aligning two sequences  $V = (v_1v_2...v_n)$  and  $W = (w_1w_2...w_m)$ .

Can we use Brute-Force method to create all the possible alignment, and then find the alignment with highest similarity score?

#### Efficient way to find a best alignment

Consider aligning two sequences  $V = (v_1v_2...v_n)$  and  $W = (w_1w_2...w_m)$ .

Can we use Brute-Force method to create all the possible alignment, and then find the alignment with highest similarity score?

This takes exponential time!

#### Efficient way to find a best alignment

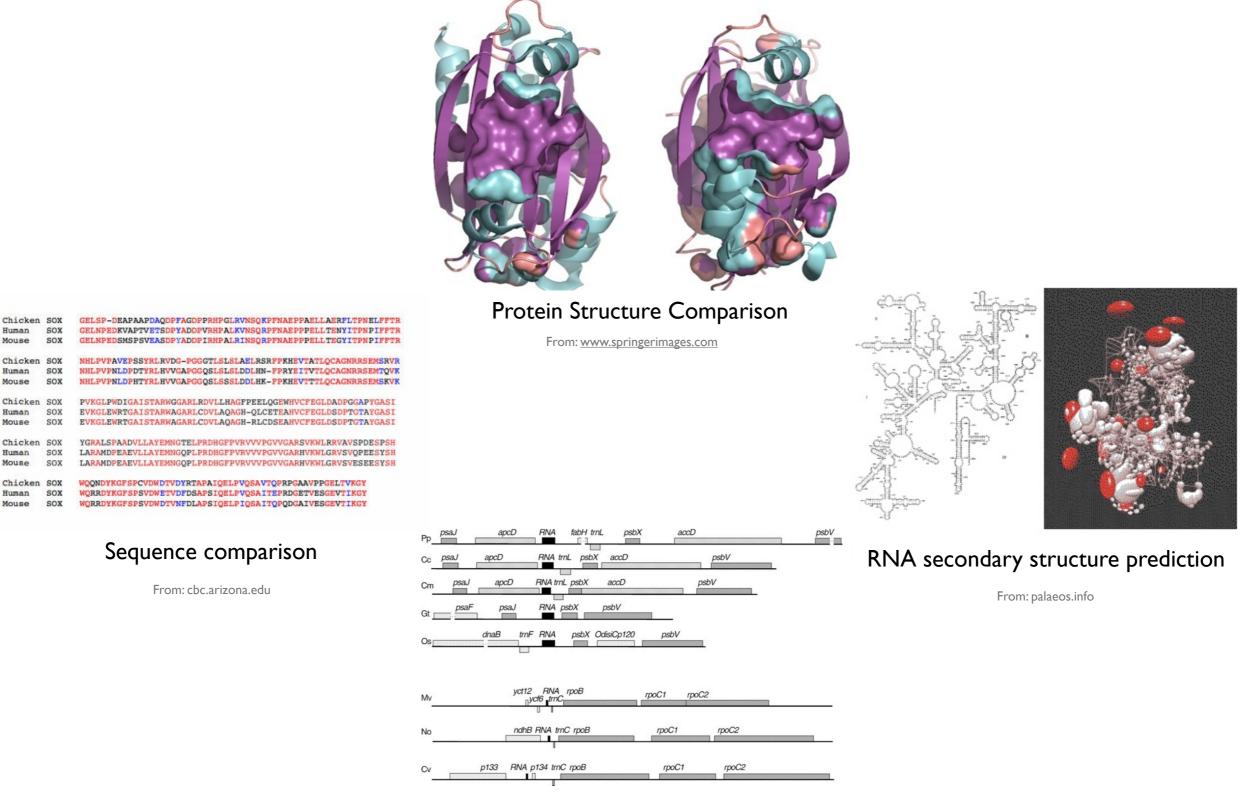
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Can we use Brute-Force method to create all the possible alignment, and then find the alignment with highest similarity score?

This takes exponential time!

**Dynamic Programming finds the optimal (best) alignment efficiently.** 

#### Problems Solvable by Dynamic Programming



#### Gene Recognition

From: pcp.oxfordjournals.org

## Dynamic Programming (DP)

# A method for efficiently solving optimization problems which have overlapping subproblems

## Property of DP problems

- Have overlapping subproblems
- Have optimal solutions to subproblems
- Can be represented in recurrence relation
- Are context-independent
  - e.g. In sequence alignment, quantifying similarity is only based on pairs of residues. Similarity is independent of environment of residues we align.

I. Global Alignment

I. Global Alignment

Best match in the entire sequences

 A
 T
 A
 C
 A
 G
 C
 G
 G
 T
 C
 T

 A
 C
 A
 G
 G
 G
 T
 C
 T

I. Global Alignment

Best match in the entire sequences

 A
 T
 A
 C
 A
 G
 C
 G
 G
 T
 C
 T

 A
 C
 A
 G
 G
 G
 T
 C
 T

II. Local Alignment

I. Global Alignment

Best match in the entire sequences

Α	Т	Α	С	Α	G	С	G	G	Т	С	Т
Α	-	-	С	Α	G	-	G	G	Т	-	Т

#### II. Local Alignment

Best subsequence match A T A C A G C G G T - C T - A C A G - G G T T - -

I. Global Alignment

Best match in the entire sequences

Α	Т	Α	С	Α	G	С	G	G	Т	С	Т
Α	-	-	С	Α	G	-	G	G	Т	-	Т

II. Local Alignment

Best subsequence match A T A C A G C G G T - C T - A C A G - G G T - - -

#### III. Semi-Global Alignment

"Glocal" Alignment

I. Global Alignment

Best match in the entire sequences

Α	Т	Α	С	Α	G	С	G	G	Т	С	Т
Α	-	-	С	Α	G	-	G	G	Т	-	Т

#### II. Local Alignment

Best subsequence match A T A C A G C G G T - C T - A C A G - G G T - - -

#### III. Semi-Global Alignment

"Glocal" Alignment

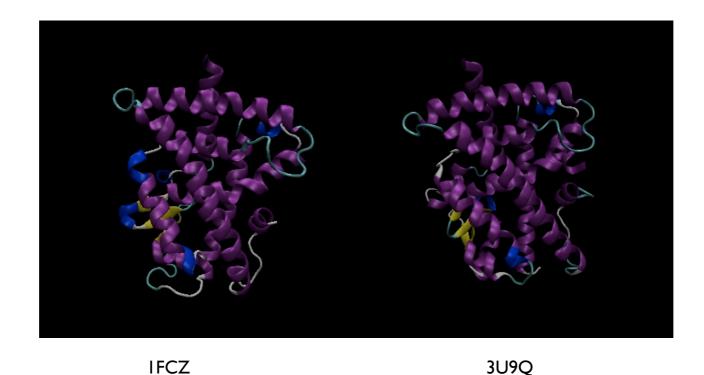
Best match without penalizing gaps on the ends of the alignment

#### Global Alignment

spans all the residues in the sequences and finds best match in the entire two sequences

- Assumes the sequences are similar over the length of one another
- The alignment attempts to match them to each other from end to end

#### IFCZ: S PQ L E E L I T K V S K A H Q E T F P - - - - S L C Q L G K - -3U9Q: S A D L R A L A K H L Y D S Y I K S F P L T K A K A R A I L T G K T T



\*\*\*\*\*\*\*\*

Optimal global alignments are produced using Needleman-Wunsch Algorithm

A dynamic programming algorithm for optimal global alignment

Given:

```
Two sequences V = (v_1v_2...v_n) and W = (w_1w_2...w_m).
(|V| = n and |W| = m)
```

Goal:

Find the best scoring alignment in which all residues of both sequences are included. The score is usually a measure of similarity.

**Requirement:** 

- A matrix NW of optimal scores of subsequence alignments. NW has size (n+1)x(m+1).
- Scoring matrix
- Defined gap penalty

#### Scoring matrix

represents a specific model of similarity to be applied in aligning two residues

- Matrix of numbers that quantify the similarity between residues
- To produce good alignment, the choice of a right scoring matrix is important
- Common scoring matrices:
  - Identity Matrix
  - Genetic Code Matrix
  - PAM Matrices
  - **BLOSUM** Matrices
- Protein sequences are frequently aligned using PAM or BLOSUM matrices that reflect the frequency with which a amino acid replaces another amino acid in evolutionarily related sequences.

Some amino acid substitutions are commonly found throughout the process of molecular evolution while others are rare.
e.g. the probability that Ser mutates into Phe is ~ three times greater than the probability that Trp mutates into Phe

	С	S	Т	Ρ	A	G	Ν	D	Е	Q	н	R	к	M	1	L	V	F	Y	W
С	9			1																
s	-1	4																		
т	-1	1	4																	-
P	-3	-1	1	7																
A	0	1	0	-1	4															
G	-3	0	-2	-2	0	6											-			
N	-3	1	0	-2	-2	0	6						1							
D	-3	0	-1	-1	-2	-1	1	6				Τ	he	sco	re	for	ali	gnir	ng a	
E	-4	0	-1	-1	-1	-2	0	2	5		1	sir	gle	ра	ir c	of a	mir	lo a	cid	s
Q	-3	0	-1	-1	-1	-2	0	0	2	5	K		•	-						
н	-3	-1	0	-2	-2	-2	1	-1	0	0	8									
R	-3	-1	-1	-2	-1	-2	0	-2	0	1	0	5								
к	-3	0	-1	-1	-1	-2	0	-1	1	1	-1	2	5							
М	-1	-1	-1	-2	-1	-3	-2	-3	-2	0	-2	-1	-1	5						
1	-1	-2	-1	-3	-1	-4	-3	-3	-3	-3	-3	-3	-3	1	4					
L	-1	-2	-1	-3	-1	4	-3	4	-3	-2	-3	-2	-2	2	2	4				
V	-1	-2	0	-2	0	-3	-3	-3	-2	-2	-3	-3	-2	1	3	1	4			
F	-2	-2	-2	-4	-2	-3	-3	-3	-3	-3	-1	-3	-3	0	0	0	-1	6		
Y	-2	-2	-2	-3	-2	-3	-2	-3	-2	-1	2	-2	-2	-1	-1	-1	-1	3	7	
W	-2	-3	-2	-4	-3	-2	-4	-4	-3	-2	-2	-3	-3	-1	-3	-2	-3	1	2	1

#### BLOSUM62

#### Gap Penalty

a score for gap between the residues of sequences in sequence alignment

Gaps inserted in a sequence to maximize similarity with another, require a scoring penalty.

Gap opening penalty: penalty for starting a new gap in a sequence.

Gap extension penalty: penalty for adding gaps to an existing gap.

Common Gap Models:

- **Constant gap:** g = (gap opening penalty)
- Linear gap:  $g(n_{gap}) = -n_{gap}$ . (gap extension penalty)
- Affine gap:  $w(n_{gap}) = -(gap opening penalty) [n_{gap}.(gap extension penalty)] = g + g(n_{gap})$

Affine gap model is used extensively in biology domain.

#### Needleman-Wunsch Algorithm (Cont.)

A dynamic programming algorithm for optimal global alignment

Given:

```
Two sequences V = (v_1v_2...v_n) and W = (w_1w_2...w_m).
(|V| = n and |W| = m)
```

Goal:

Find the best scoring alignment in which all residues of both sequences are included. The score is usually a measure of similarity.

**Requirement:** 

- A matrix NW of optimal scores of subsequence alignments. NW has size (n+1)x(m+1).
- Score matrix
- Defined gap penalty

#### Needleman-Wunsch Algorithm (Cont.) Calculation

Let NW(i,j) be the optimal alignment score of aligning V[1...i] and W[1...j]

		WI	•••	wj	•••	Wm
	0					
٧I				(111)		
:			(I) +s(v	i, w <sub>j</sub> ) +gap		
Vi			(II) <mark>_</mark> +ga			
•						
Vn						Optimal alignment score

#### Needleman-Wunsch Algorithm (Cont.) Calculation

Let NW(i,j) be the optimal alignment score of aligning V[1...i] and W[1...j]

		WI	•••	wj	•••	Wm
	0					
٧I				(111)		
:			(I) +s(v	i, w <sub>j</sub> )_+gap		
Vi			(II)_ <mark>+ga</mark>	$P \xrightarrow{i} V$		
•						
Vn						Optimal alignment score

Base case:  $\begin{cases} NW(0,0) = 0 & \text{For linear gap penalty model} \\ NW(0,j) = NW(0,j-1) + g & \text{operations:} \\ NW(i,0) = NW(i-1,0) + g & \text{operations:} \end{cases}$ Recurrence:  $NW(i,j) = \max \begin{cases} NW(i-1,j-1) + s(v_i,w_j) & \text{match/mismatch} \\ NW(i-1,j) + g & \text{delete} \\ NW(i,j-1) + g & \text{insert} \end{cases}$ 

#### Dynamic Programming Approach Summary

Construct an optimal alignment between two subsequences  $(v_1v_2...v_i)$  and  $(w_1w_2...w_j)$ , (Where  $0 \le i \le n$  and  $0 \le j \le m$ ), by considering the three cases:

(I) The optimal alignment of  $v_{1,...,v_{i-1}}$  with  $w_{1,...,w_{j-1}}$ , extended by the match between  $v_i$  and  $w_{j.}$ .

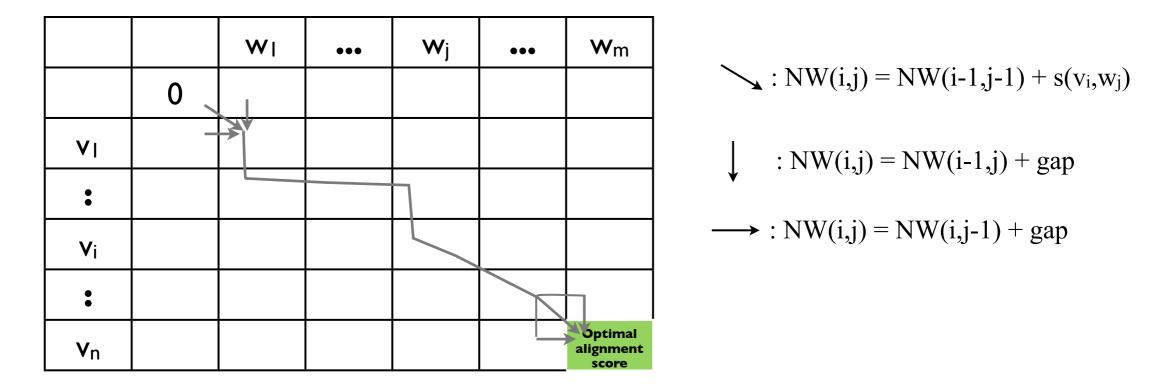
(II)The optimal alignment of  $v_{1,...,v_{i-1}}$  with  $w_{1,...,w_{j}}$ , extended by matching a gap character with  $v_i$ .

(III)The optimal alignment of  $v_{1,...,v_i}$  with  $w_{1,...,w_{j-1}}$ , extended by matching  $w_j$  with a gap character "-".

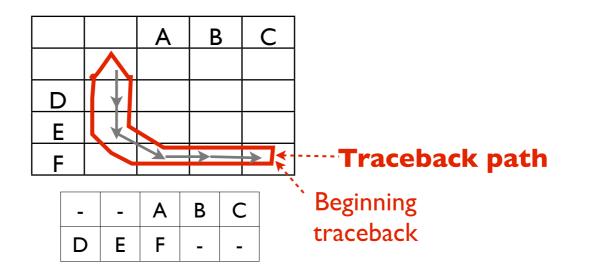
Store these optimal scores of subsequence alignments in a matrix of size  $(n+1) \times (m+1)$ .

# Needleman-Wunsch Algorithm (Cont.)

To recover the optimal alignment, arrows indicating forward calculation paths, are placed in each entry.



#### Determine alignment from the end of the sequences



# Needleman-Wunsch Algorithm (Cont.)

Optimal global alignment of V = THISLINE and W= ISALIGNED with gap =  $-4n_{gap}$ , score matrix = BLOSUM62

		Ι	S	A	L	Ι	G	N	E	D
	0	→-4-	<b>→</b> -8 -	-12→	→-16-	►20 -	→24-	-28−	-32-	<b>→</b> 36
Т	<b>↓</b> _4.	-1	-3 -	→-7-	→-11-	→-15	<b>→</b> -19_	-23-	≁27-	→-31
Η	⊻8	-5	<b>▲</b> -2	-5	-9	-13	-17-	-18	-22	-26
Ι	-12	-4	-6	-3	-3	-5	-9	-13	-17	-21
S	-16	-8	× 0 =	4	-5	-5	-5	-8	-12	-16
L	-20	-12	-4	-1	× 0	-3	-7	-8	-11	-15
Ι	-24	-16	-8	-5	1	4_	0	-4	-8	-12
N	-28	-20	-12	-9	-3	0	4	× Ó	2	-2
E	-32	-24	-16	-13	-7	-4	0	4	×11_	7

From: Understanding Bioinformatics by Zvelebil, Baum

V:	Т	Н	I	S	-	L	I	-	Ν	E	-
W:	-	-	I	S	А	L	I	G	Ν	Е	D

#### Needleman-Wunsch Algorithm Efficiency

Runtime: O(nm)

Why?

Space: O(nm)

for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

		WI		Wj	•••	Wm
	0			4	-(i)	
VI				' ('-')	aps	
:			+s(*	gaps w <sub>i</sub> )+gap		
Vi	<u>+ (j) gap</u>	+(j-1	)gap <u>+ga</u> r			
:						
Vn						Optimal alignment score

for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

		WI		Wj	•••	Wm
	0			+	(i)	
٧ı				' ('-')	aps	
•			+s(+	gaps w <sub>i</sub> )+gap		
Vi	<u>+ (j) gap</u>	+(j- l	)gap <u>+ga</u> r			
•						
Vn						Optimal alignment score

$$\begin{split} NW(i,j) &= \max \begin{cases} NW(i-1,j-1) + s(v_i,w_j) & \text{match/mismatch} \\ [NW(i-n_{gap},j) + w(n_{gap1})]_{1 \leq n_{gap1} \leq i} & \text{delete} \\ [NW(i,j-n_{gap2}) + w(n_{gap2})]_{1 \leq n_{gap2} \leq j} & \text{insert} \end{cases} \end{split}$$

for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

		WI		Wj	•••	Wm
	0			-	(i)	
٧I				+ (i-1) <sup>g</sup>	aps	
•			+5(*	w <sub>i</sub> )+gap		
Vi	<u>+ (j) gap</u>	+(j-	)gap <u>+ga</u> r			
•						
Vn						Optimal alignment score

$$\begin{split} NW(i,j) &= \max \begin{cases} NW(i-1,j-1) + s(v_i,w_j) & \text{match/mismatch} \\ [NW(i-n_{gap},j) + w(n_{gap1})]_{1 \le n_{gap1} \le i} & \text{delete} \\ [NW(i,j-n_{gap2}) + w(n_{gap2})]_{1 \le n_{gap2} \le j} & \text{insert} \end{cases} \end{split}$$

What is the runtime? or space?

#### Needleman-Wunsch Algorithm

for any gap penalty models

Does affine gap model work with simple Needleman-Wunsch Algorithm we just saw?

		WI		Wj		Wm
	0			-	(i)	
٧ı				+ (i-1) <sup>g</sup>	aps	
•			+5(*			
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•						
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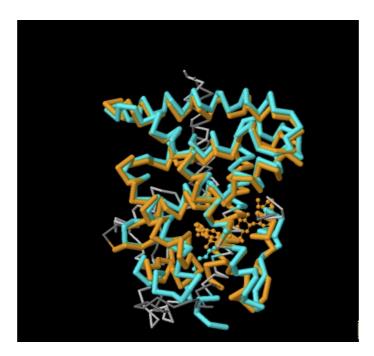
What is the runtime? or space?  $O(mn^2)$  where n>m

#### Local Alignment

finds the most similar regions of a nucleotide or amino acid sequence ignoring other segments of the sequences

Local alignment programs are useful for detecting shared domains in multi-domain proteins.

IFCZ: ATKCIIKIVEFAKRLPGFTGLSIAACLDILMLRIC 3U9Q: SVEAVQEITEYAKSIPGFVNLDLNDQVTLLKYGVH



IFCZ and 3U9Q superimposed

Optimal local alignments are produced using Smith-Waterman Algorithm

#### Smith-Waterman Algorithm

A dynamic programming algorithm for optimal local alignment

Given:

```
Two sequences V = (v_1v_2...v_n) and W = (w_1w_2...w_m).
(|V| = n and |W| = m)
```

Goal:

Find the highest scoring alignment for best subsequence match. The score is usually a measure of similarity.

Requirement:

- A matrix SW of optimal scores of subsequence alignments. SW has size (n+1)x(m+1).
- Score matrix
- Defined gap penalty

		WI	•••	wj	•••	Wm
	0	0	0	0	0	0
VI	0			(111)		
:	0		(I) +s(v	(iii) i, w <sub>j</sub> )_+gap		
Vi	0		(II) <mark>_</mark> +ga			
:	0					
Vn	0					

		WI	•••	wj	•••	Wm
	0	0	0	0	0	0
VI	0			(   )		
:	0		(I) +s(v	(iii) i, w <sub>j</sub> )_+gap		
Vi	0		(II) <mark>_</mark> +ga			
:	0					
Vn	0					

**Optimal alignment score =**  $\max_{0 \le i \le n, 0 \le j \le m} \{SW(i, j)\}$ 

		WI	•••	Wj	•••	Wm
	0	0	0	0	0	0
VI	0			(   )		
:	0		(I) +s(v	i, w <sub>j</sub> )_+gap		
Vi	0		(II) <mark>_</mark> +ga			
:	0					
Vn	0					

**Optimal alignment score =**  $\max_{0 \le i \le n, 0 \le j \le m} \{SW(i, j)\}$ 

For linear gap penalty model

Base case: SW(i,j) = 0 where i = 0 or j = 0

$$\mbox{Recurrence:} \quad SW(i,j) = \max \begin{cases} 0 & \mbox{align empty strings} \\ SW(i-1,j-1) + s(v_i,w_j) & \mbox{match/mismatch} \\ SW(i-1,j) + g & \mbox{delete} \\ SW(i,j-1) + g & \mbox{insert} \end{cases}$$

Local alignment of V = THISLINE , W= ISALIGNED with gap =  $-4n_{gap}$  , score matrix = BLOSUM62

		Ι	S	A	L	Ι	G	N	E	D
	0	0	0	0	0	0	0	0	0	0
Τ	0	0	1	0	0	0	0	0	0	0
Η	0	0	0	0	0	0	0	1	0	0
Ι	0	4	0	0	2	4	0	0	0	0
S	0	0	8 -	-4	0	0	4	1	0	0
L	0	2	4	7	8	4	0	1	0	0
Ι	0	4	0	3	9	12-	→ O	4	0	0
Ν	0	0	5	1	5	8	12	14	10	6
E	0	0	1	4	1	4	8	12	19-	→15

From: Understanding Bioinformatics by Zvelebil, Baum

#### Traceback starts at entry containing the optimal alignment score.

V:	I	S	-	L	I	-	N	E
W:	I	S	Α	L		G	N	E

#### Smith-Waterman Algorithm Efficiency

Runtime: O(nm)

Why?

Space: O(nm)

Example I. Overlap detection: Aligning exon to a gene sequence V = ATCCGAACATCCAATCGAAGC W = AGCATGCAAT

Aligning scores: match = 2, gap = -1 mismatch = -2

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Global alignment?

A T C C G A A C A T C C A A T C G A A G C A - - G - - C A T G C A A T - - - - - -Score = 2(9) - I (II) - 2(I) = 5

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Local alignment?

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Aligning scores: match = 2, gap = -1 mismatch = -2

Global alignment?A T C C G A A C A T C C A A T C G A A G C<br/>A - - - G - - C A T G C A A T - - - - -<br/>Score = 2(9) - I(II) - 2(I) = 5Local alignment?C A T C C A A T<br/>C A T G C A A T<br/>Score = 2(7) - 2(I) = 12

Example I. Overlap detection: Aligning exon to a gene sequence V = ATCCGAACATCCAATCGAAGC W = AGCATGCAAT

Aligning scores: match = 2, gap = -1 mismatch = -2

Global alignment?A T C C G A A C A T C C A A T C G A A G C<br/>A - - - G - - C A T G C A A T - - - -<br/>Score = 2(9) - I(11) - 2(1) = 5Local alignment?C A T C C A A T<br/>C A T G C A A T<br/>Score = 2(7) - 2(1) = 12

Where was the overlap exactly?

example I continued

What if avoid penalizing the gaps at the beginning and /or the end of an alignment?

Spaces in front or end of the exon might be UTR, introns, or enhancer and etc. Thus these gaps should not be penalized.

example I continued

What if avoid penalizing the gaps at the beginning and /or the end of an alignment?

Spaces in front or end of the exon might be UTR, introns, or enhancer and etc. Thus these gaps should not be penalized.

**Semi-global alignment**. Globally aligning the two sequence but ignoring penalizing gaps at both ends of a sequence.

#### Which alignment to use? (Cont.)

Example 2. Overlap detection: Sequence assembly: V = ACCTCACGATCCGA W = TCAACGATCACCGCA

#### - - - - - - ACCTCACGATCCGA TCAACGATCACCGCA - - - - - - - -

**Semi-global alignment**. Globally aligning the two sequence but ignoring penalizing the starting gaps of a sequence and the trailing gaps of the other sequence.

#### Semi-Global Alignment

finds optimal alignment without penalizing gaps on the ends of the alignment

How to perform semi-global alignment?

#### Modify the basic Needleman-Wunsch algorithm:

Set the first row and first column of the DP matrix to 0.

		WI	•••	Wj	•••	Wm
	0	0	0	0	0	0
VI	0			(111)		
•	0		(I) +s(v	(III) i, w <sub>j</sub> )_+gap		
Vi	0		(II) <mark>_</mark> +ga			
:	0					
Vn	0					

**Optimal alignment score = max ( row**<sub>n</sub>, column<sub>m</sub>)

Traceback starts at entry containing the optimal alignment score and ends at the first row or the first column.

### Versatility of DP Algorithm

- Memory usage can be optimized
- Runtime can be improved

## Versatility of DP Algorithm

- Memory usage can be optimized
- Runtime can be improved
- Heuristically can improve the runtime:
  - FASTA
  - BLAST

#### References

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### Any Question?